PHYSICAL REVIEW B **71**, 220506(R) (2005)

Switching superconductivity in superconductor/ferromagnet bilayers by multiple-domain structures

T. Champel and M. Eschrig
Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany

(Received 12 April 2005; published 13 June 2005)

We consider the effect of a multiple magnetic domain structure in a superconductor/ferromagnet bilayer, modeled by a ferromagnetic layer with a rotating magnetic moment. The domain walls in this model are of equal size as the domains, and are of Néel type. We study the superconducting critical temperature as a function of the rotation wavelength of the magnetic moment. The critical temperature of the bilayer is found to be always enhanced by the domain structure, and exhibits an interesting reentrant behavior. We suggest that this effect can be used for a new device where superconductivity may be controlled by the domain structure of the magnetic layer.

DOI: 10.1103/PhysRevB.71.220506 PACS number(s): 74.45.+c, 74.62.−c, 74.78.−w

The study of the proximity effect between a superconductor (S) and a ferromagnet (F) is currently a very active field due to its relevance for nanoelectronic applications and due to the perspectives to discover new interesting physical phenomena. It is well-known that the exchange field **J** of the ferromagnet tends to break the Cooper pairs formed by electrons with opposite spins by acting on the electronic spins via the exchange interaction. When the exchange field exceeds the typical superconducting low-energy scales, $J > 2\pi T_{c0}$ (T_{c0} is the superconducting critical temperature in absence of the proximity effect), a new, shorter length scale competes with the superconducting coherence length scale. For this case, within the quasi-classical theory of superconductivity for diffusive structures (Usadel equations[1]), the superconducting pair correlations have been found to penetrate (and also oscillate) in the F part over a short length $\xi_J = \sqrt{D_f/J}$ instead of the superconducting coherence length $\xi_f = \sqrt{D_f/2\pi T_{c0}}$ (D_f is the diffusion constant in F).[2] Moreover, in S/F bilayers the critical temperature T_c is suppressed by the proximity of the F layer and exhibits a non-monotonic dependence on the thickness of the F layer, d_f (see Ref. 3 and references therein). In F/S/F sandwiches, it has been found that the pair-breaking effect depends on the relative orientation of the F moments. This property led to the proposal of a superconducting switch[4,5] operated by reversing the moment in a F layer.

In this paper, we introduce another aspect of this feature. We investigate the sensitivity of the Cooper pairs on the directional changes of the exchange field by studying a S/F bilayer with a moment rotating with a constant velocity in the F layer[6] (see Fig. 1). This model simulates a multidomain structure with domain walls of the Néel type, where the domain wall width is comparable in size to the size of the domains. We find that the critical temperature T_c is enhanced compared to that of the S/F bilayer with homogeneous magnetization and exhibits an interesting reentrant behavior. We suggest to use this effect for new devices where superconductivity may be switched on and off by controlling the magnetic domain structure.

Recently, the theoretical understanding of the interplay between the superconductivity and inhomogeneous magnetism has been improved by realizing that a local inhomogeneous exchange field induces superconducting triplet correlations, which penetrate over a long scale, ξ_f, in a diffusive ferromagnet.[7,8] As a result, these correlations lead to a significant increase of the conductance of the ferromagnet below T_c.[7] Volkov *et al.*[9,10] have demonstrated the existence of these long-range triplet components (TC) also in the multilayered F/S/F structures when the moments in each F layers are non-collinear. As already emphasized in Refs. 7, 9, and 10, these TC have the property to be odd in frequency, a necessary condition for their existence in the diffusive limit. The study of T_c as a function of the angle between the moments in the F/S/F trilayer has been reported very recently.[11] In order to clarify the situation and to compare with our model, we first derive generally the Usadel equations in the paired-spin basis and show that the coexistence of singlet components (SC) with odd-triplet components is in fact the hallmark of any diffusive S/F structure.[12] Short-range TC are necessarily present. Contrarily, the presence of TC with long-range scales requires specific conditions. For example, the long-range TC are shown to be absent within our model. The sensitivity of T_c on the magnetic inhomogeneity is in our model mainly provided by the short-range TC.

We study the S/F bilayer within the framework of the Nambu-Gor'kov formalism and consider the diffusive limit which is relevant for short mean free paths. The Usadel Green's function $\hat{g}(\mathbf{R}, \omega_n)$ depends on the spatial coordinate $\mathbf{R}$ and on the Matsubara frequency $\omega_n = \pi T(2n+1)$ (where T

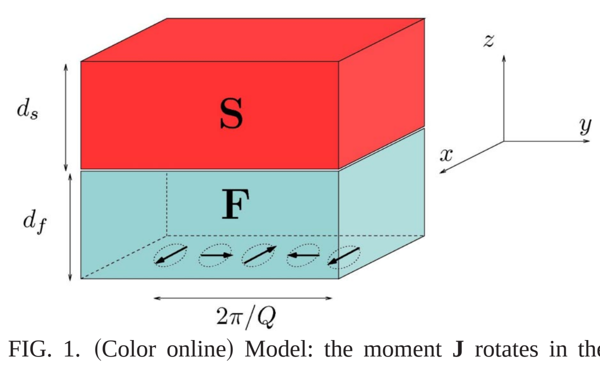

FIG. 1. (Color online) Model: the moment **J** rotates in the F layer.

is the temperature), and results from the momentum average of the quasi-classical Green's function. It is represented by a 4×4 Nambu-Gor'kov matrix in combined particle-hole and spin spaces. In particle-hole space it is written as

$$\hat{g}=\begin{pmatrix} g & f \\ f^{\dagger} & g^{\dagger}\end{pmatrix}, \tag{1}$$

where the 2×2 spin-matrices g and f represent the normal and anomalous Green's functions. $\hat{g}$ obeys a normalization condition and a nonlinear transport equation, coupling its components f and g.[1] Near T_c, f is small and g deviates slightly from its value $(-i\pi\,\mathrm{sgn}\,\omega_n)$ in the normal state, so that the Usadel transport equation can be linearized with the help of the normalization condition, and yields an equation for the 2×2 spin-matrix f

$$D\boldsymbol{\nabla}^2 f - i\,\mathrm{sgn}(\omega_n)\{\mathbf{J}(\mathbf{R})\cdot\boldsymbol{\sigma} f + f\mathbf{J}(\mathbf{R})\cdot\boldsymbol{\sigma}^*\} - 2|\omega_n| f + 2\pi\Delta = 0 \tag{2}$$

with $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ a vector whose components are the Pauli matrices. In the F layer the superconducting order parameter Δ vanishes, and in the S layer the exchange field $\mathbf{J}$ is zero. The diffusion constants are $D=D_s$ in S and $D=D_f$ in F. We consider a BCS singlet superconductor for which the order parameter in spin-space is $\Delta=\Delta_s i\sigma_y$. Quite generally, f is expected to have a singlet and a triplet part[13]

$$f = f_s i\sigma_y + i(\mathbf{f}_t\cdot\boldsymbol{\sigma})\sigma_y, \tag{3}$$

where $\mathbf{f}_t=(f_{tx},f_{ty},f_{tz})$ is the triplet vector (the basis formed by the vectors $s=(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)/2$, $t_x=(|\downarrow\downarrow\rangle-|\uparrow\uparrow\rangle)/2$, $t_y=(|\uparrow\uparrow\rangle+|\downarrow\downarrow\rangle)/2i$, and $t_z=(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)/2$ is the natural basis for studying pairing states). The different components of f obey the system of coupled equations

$$(D\boldsymbol{\nabla}^2 - 2|\omega_n|)f_s = -2\pi\Delta_s + 2i\,\mathrm{sgn}(\omega_n)\mathbf{J}\cdot\mathbf{f}_t, \tag{4}$$

$$(D\boldsymbol{\nabla}^2 - 2|\omega_n|)\mathbf{f}_t = 2i\,\mathrm{sgn}(\omega_n)\mathbf{J}f_s. \tag{5}$$

The couplings (right-hand side) in Eqs. (4) and (5) show obviously that the SC f_s always coexists with at least one TC as early as $\mathbf{J}\neq\mathbf{0}$. We see also generally that the SC is even in frequency $[f_s(-\omega_n)=f_s(\omega_n)]$ as it is the case for the BCS superconductors, while the TC are odd $[\mathbf{f}_t(-\omega_n)=-\mathbf{f}_t(\omega_n)]$. This is a consequence of the Pauli principle, which for the diffusive limit imposes the relation $f_{\alpha\beta}(\mathbf{R},\omega_n)=-f_{\beta\alpha}(\mathbf{R},-\omega_n)$, where α and β are the spin indices. Henceforth, we only consider positive Matsubara frequencies. Equations (4) and (5) are supplemented by the self-consistent equation relating Δ_s to f_s

$$\Delta_s \ln\frac{T_{c0}}{T} = 2\pi T\sum_{n\geqslant 0}\left(\frac{\Delta_s}{\omega_n}-\frac{f_s(\omega_n)}{\pi}\right), \tag{6}$$

and also by the boundary conditions. The general boundary conditions at the S/F interface for the diffusive case have been formulated by Nazarov[14] and reduce near T_c to

$$\xi_s\partial_z f)_S = \gamma\xi_f\partial_z f)_F, \quad \gamma=\rho_s\xi_s/\rho_f\xi_f \tag{7}$$

where ρ_s and ρ_f are, respectively, the normal-state resistivities of the S and F metals [Eq. (7) follows from the continuity of the current at the interface], and

$$\xi_f\gamma_b\partial_z f)_F = f)_S - f)_F, \quad \gamma_b = R_b\mathcal{A}/\rho_f\xi_f \tag{8}$$

with R_b the resistance of the S/F boundary, and $\mathcal{A}$ its area. Here z denotes the distance to the S/F interface. Note that near T_c Nazarov's boundary conditions are formally equivalent to the ones by Kuprianov and Lukichev.[15] At the outer surfaces of the F or S layers, we require that the current through the boundary has to vanish. It is important to note that the conditions (7) and (8) do not couple the different components of f.

If $\mathbf{J}$ is constant in direction, we obtain straightforwardly that the triplet vector $\mathbf{f}_t\parallel\mathbf{J}$. This configuration corresponds to a triplet state with a zero spin projection on the quantization axis defined by $\hat{\mathbf{J}}$. The SC and the TC with zero spin projection are energetically equivalent with respect to the exchange interaction, and thus necessarily appear together in the ferromagnet.

In F/S/F trilayers with an arbitrary angle between the moments in each F layer,[9–11] $\mathbf{J}$ has not a fixed direction in the structure. If the moments are homogeneous in each F layer, the system (4) and (5) can be solved easily. In the different F layers, the spatial dependence of the components of f can be written as $f_i(z)=f_i\exp(k_f z)$, leading to the algebraic equations

$$\begin{pmatrix} D_f k_f^2 - 2\omega_n & -2i\,\mathbf{J}\cdot \\ -2i\,\mathbf{J} & D_f k_f^2 - 2\omega_n\end{pmatrix}\begin{pmatrix} f_s \\ \mathbf{f}_t\end{pmatrix} = 0. \tag{9}$$

The eigenvalue k_f^2 is determined from the condition of zero determinant, which yields

$$k_{f\pm}^2 = \Omega_n\xi_f^{-2} \pm 2i\xi_J^{-2} \text{ or } k_{f0}^2 = \Omega_n\xi_f^{-2} \tag{10}$$

with $\Omega_n=\omega_n/\pi T_{c0}$. The corresponding eigenvectors have the form

$$\begin{pmatrix} f_{s\pm} \\ \mathbf{f}_{t\pm}\end{pmatrix} = f_{s\pm}\begin{pmatrix} 1 \\ \pm\hat{\mathbf{J}}\end{pmatrix} \text{ and } \begin{pmatrix} f_{s0} \\ \mathbf{f}_{t0}\end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{f}_{t0}\end{pmatrix} \tag{11}$$

with the condition $\mathbf{f}_{t0}\cdot\mathbf{J}=0$. Thus, there can exist *a priori* two very different types of TC.[7,8] The SC f_s and the TC $\parallel\mathbf{J}$ characterized by $k_{f\pm}^2$ have the usual short-range decay length ξ_J in the F layer for strong J. On the contrary, the TC $\mathbf{f}_t\perp\mathbf{J}$ (characterized by k_{f0}^2) penetrate further in the ferromagnet. Since they correspond (locally) to triplet Cooper pairs with a nonzero spin projection (equal-spin pairing), they are not locally limited by the paramagnetic interaction with $\mathbf{J}$.

Now, we consider the S/F bilayer shown in Fig. 1 where $\mathbf{J}$ rotates with a constant velocity Q in the F layer, i.e., $\mathbf{J}(y)=J(\cos Qy,\sin Qy,0)$ [henceforth z no longer designates the quantization axis]. It is straightforward to see that here $f_{tz}=0$. The y dependence of $\mathbf{J}$ is eliminated in the right-hand side of Eqs. (4) and (5) by considering the new components $f_+=(-f_{tx}+if_{ty})e^{iQy}$ and $f_-=(f_{tx}+if_{ty})e^{-iQy}$. The system of equations to solve is

$$(D\boldsymbol{\nabla}^2 - 2\omega_n)f_s = -2\pi\Delta_s + i\,J(f_- - f_+), \tag{12}$$

$$(D\nabla^2 \mp 2iDQ\partial_y - DQ^2 - 2\omega_n)f_\pm = \mp 2iJf_s. \qquad (13)$$

Since the structure is periodical in the y direction, the three components f_l $(l=s,\pm)$ and Δ_s are expanded into Fourier series. We note that the different Fourier components (labeled by p) are neither mixed by the full system of linearized equations nor by the boundary conditions. Thus, each harmonic taken separately (and determining a particular y dependence) represents a possible solution. The harmonic p realized physically is the one which gives the highest T_c. As a result of our calculations,[16] the harmonic $p=0$ yields the highest T_c. The physical reason for this is that the other harmonic solutions correspond to an inhomogeneous bulk superconducting state, which decreases unavoidably T_c. We present here the equations for the case $p=0$.

In the F layer, using the boundary condition at the outer surface $(z=-d_f)$, the components f_l are thus sought under the form $f_l(z)=f_l\cosh[k_f(z+d_f)]$, which is substituted in the set of Eqs. (12) and (13). This leads to the following linear system

$$\begin{pmatrix} \lambda & -i\xi_J^{-2} & i\xi_J^{-2} \\ -2i\xi_J^{-2} & \lambda - Q^2 & 0 \\ 2i\xi_J^{-2} & 0 & \lambda - Q^2 \end{pmatrix}\begin{pmatrix} f_s \\ f_- \\ f_+ \end{pmatrix} = 0 \qquad (14)$$

with the eigenvalue $\lambda = k_f^2 - \Omega_n\xi_f^{-2}$. This system yields the three eigenvalues (and consequently three k_f)

$$\lambda_\varepsilon = (Q^2 + \varepsilon\sqrt{Q^4 - 16\xi_J^{-4}})/2, \quad \lambda_0 = Q^2 \qquad (15)$$

and the associated eigenvectors (here $\varepsilon = \pm 1$)

$$\begin{pmatrix} f_{s,\varepsilon} \\ f_{-,\varepsilon} \\ f_{+,\varepsilon} \end{pmatrix} = \begin{pmatrix} \lambda_{-\varepsilon} \\ -2i\xi_J^{-2} \\ 2i\xi_J^{-2} \end{pmatrix}, \quad \begin{pmatrix} f_{s,0} \\ f_{-,0} \\ f_{+,0} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}. \qquad (16)$$

The eigenvalues λ_ε give short-range decay lengths for the TC in strong ferromagnets, while the eigenvalue λ_0 determines a long-range penetration in the ferromagnet (at least at small Q). The solution of Eqs. (12) and (13) independent of y is thus written as

$$f_l(z) = \sum_{j=0,\varepsilon} a_j f_{l,j}\cosh[k_{fj}(z+d_f)], \qquad (17)$$

where the three coefficients a_j have to be determined with the help of the boundary conditions at the S/F interface.

In the S layer, the solutions for the TC $f_\pm$ satisfying the boundary condition at the outer surface (at $z=d_s$) are straightforwardly derived $f_\pm(y,z)=c_\pm\cosh[k_s(z-d_s)]$ where $k_s=\sqrt{\Omega_n\xi_s^{-2}+Q^2}$, $\xi_s=\sqrt{D_s/2\pi T_{c0}}$, and $c_\pm$ are coefficients. As a result of the form of the eigenvector associated with the eigenvalue λ_0, the boundary conditions for the TC at the S/F interface require $a_0=0$. This absence of the long-range TC in the present model is characterized by the fact that the triplet vector $\mathbf{f}_t(y,z)$ is locally parallel to $\mathbf{J}(y)$.

The remaining equation for the SC $f_s(z)$ can generally not be solved analytically in the S layer since it is coupled with the self-consistent gap $\Delta_s(z)$ by Eq. (6). We use a technical high-energy cutoff of $\omega_n=21\pi T_{c0}$ for the gap equation. We iterate Eqs. (6) and (12) in S numerically, together with the boundary conditions (7) and (8), which determine the coefficients a_j and $c_\pm$.

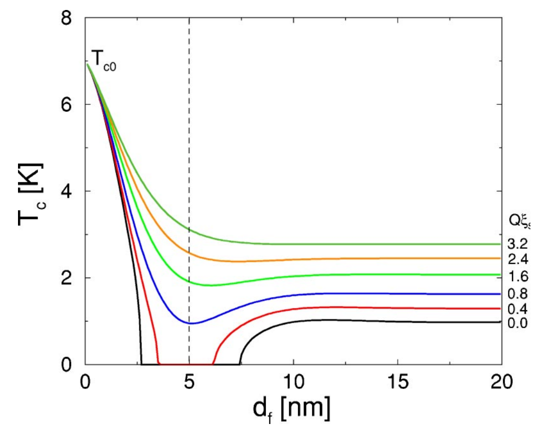

FIG. 2. (Color online) T_c vs d_f for various $Q\xi_s$. Here $d_s=7$ nm. For $d_f\approx 5$ nm, corresponding to the dashed line, a switching effect can be observed between $Q=0$ (single domain F) and $Q\xi_s>0.56$.

In Fig. 2 we show the dependence of T_c on d_f for different values of the dimensionless parameter $Q\xi_s$ (for definiteness we took the same parameters as in Fig. 2 of Ref. 3: $T_{c0}=7$ K, $\gamma=0.15$, $\gamma_b=0.3$, $J=130$ K, $\xi_s=8.9$ nm, and $\xi_f=7.6$ nm). T_c clearly increases with Q. Moreover, we see that the non-monotonic behavior of $T_c(d_f)$, responsible for the absence of superconductivity in a finite intermediate range of d_f (around $d_f=5$ nm in Fig. 2), tends to be suppressed in favor of a monotonic behavior. This tendency is accompanied by a reentrance of superconductivity as a function of magnetic inhomogeneity in the d_f range where T_c is zero. Fixing d_f in this range we predict that when Q exceeds some critical value Q_{crit} $(\approx 0.56/\xi_s$ for $d_f=5$ nm), superconductivity is recovered. By controlling the degree of magnetic inhomogeneity it is possible to *switch the bilayer between the normal state and the superconducting state*.

The critical behavior is illustrated for $d_f=5$ nm in Fig. 3, where T_c is plotted as a function of $Q\xi_s$ for different d_s. It is clear that T_c is enhanced for any Q compared to the homogeneous case $Q=0$. The switching behavior can be observed

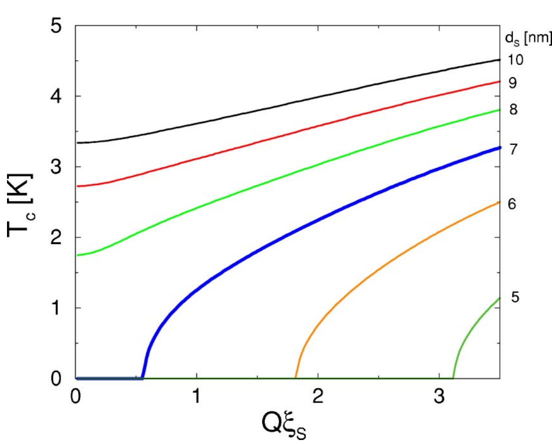

FIG. 3. (Color online) T_c vs $Q\xi_s$ for $d_f=5$ nm and various d_s. The thick curve corresponds to a cut along the dashed line in Fig. 2. The onsets for $d_s<7.44$ nm behave like $\sqrt{Q-Q_{\text{crit}}}$.

for superconducting layers with $d_s < d_{s,\mathrm{crit}}$, where for the used parameters $d_{s,\mathrm{crit}} \approx 7.44$ nm. For $d > d_{s,\mathrm{crit}}$ we observe for small Q a behavior $T_c \sim Q^2$. For $d < d_{s,\mathrm{crit}}$ there is an onset of superconductivity at Q_{crit}, and near the onset the functional behavior of T_c as function of Q is $T_c \sim \sqrt{Q - Q_{\mathrm{crit}}}$. By choosing the thickness of the superconducting layer appropriately, it is possible to optimize the superconducting switch for the corresponding range of Q in which it can be controlled.

Finally, we discuss the implications of our results for the currently debated general issue of the influence on the superconductivity of domain walls present in the F layer. The evolution of **J** in the domain walls can typically be described by two kinds of models. In bulk material, **J** rotates about an axis perpendicular to the wall, i.e., it remains in the plane of the domain wall (Bloch wall). On the contrary, in thin F layers **J** is expected to evolve in the surface plane (Néel wall). Our model can be seen as a model for a F layer containing many Néel walls whose widths are comparable to the widths of the domains. Recently, a dependence of T_c on the (controlled) domain state of the ferromagnet has been observed[17] in S/F bilayers. In the presence of many domain walls, T_c has been found to be slightly enhanced compared to the T_c obtained in the absence of domain walls. The authors of Ref. 17 argue that the domain walls in their experiment are likely of the Néel type. Indeed, the presence of Bloch walls in a F layer induces unavoidably a nonzero component for **J** in the direction perpendicular to the layer. As a result, the exchange field acting on the electron spins produces in addition an electromagnetic field acting via the electronic charges on the orbital motion of electrons and thus suppresses the superconductivity. This rising orbital pair-breaking effect competes with the weakened paramagnetic pair-breaking effect in the domain walls and is expected to reduce or even prevent any dependence of T_c on the domain state. Our results are thus consistent with a T_c enhancement due to Néel domain walls. A future step is to take into account more realistically the alternation of domain regions with domain walls. The difficulty is that the problem is intrinsically two-dimensional, and the solutions with separated y and z dependences are no more physically acceptable for boundary reasons. This prevents any simple analytical treatment even in the F layer as done here.

The possibility to reversibly create and remove Néel walls in a controlled way as needed for the realization of the suggested device has been shown very recently.[18] The method used in Ref. 18 is based on the exchange bias effect between a ferromagnetic Co thin film and an antiferromagnetic insulating CoO film.

In conclusion, we have presented a model for a multiple magnetic domain structure in an S/F bilayer and have shown that the superconducting critical temperature is enhanced by the magnetic inhomogeneity and even exhibits reentrant superconducting behavior appealing for future applications. We suggest to use this effect for a superconducting switch operated by controlling the degree of inhomogeneity in the ferromagnetic layer.

This work was supported by the Deutsche Forschungsgemeinschaft within the Center for Functional Nanostructures.

[1] K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[2] Z. Radovic *et al.*, Phys. Rev. B **44**, 759 (1991).
[3] Ya. V. Fominov *et al.*, Phys. Rev. B **66**, 014507 (2002).
[4] L. R. Tagirov, Phys. Rev. Lett. **83**, 2058 (1999).
[5] A. I. Buzdin *et al.*, Europhys. Lett. **48**, 686 (1999).
[6] The effect of the superconductivity on the magnetic order has been studied within the same model in F. S. Bergeret *et al.*, Phys. Rev. B **62**, 11872 (2000).
[7] F. S. Bergeret *et al.*, Phys. Rev. Lett. **86**, 4096 (2001).
[8] A. Kadigrobov *et al.*, Europhys. Lett. **54**, 394 (2001).
[9] A. F. Volkov *et al.*, Phys. Rev. Lett. **90**, 117006 (2003).
[10] F. S. Bergeret *et al.*, Phys. Rev. B **68**, 064513 (2003).
[11] Ya. V. Fominov *et al.*, JETP Lett. **77**, 510 (2003).
[12] This feature has already been mentioned in Ref. 10. Here, we provide a general demonstration of it.
[13] V. P. Mineev and K. V. Samokhin, *Introduction to Unconventional Superconductivity* (Gordon and Breach, New York, 1999).
[14] Yu. V. Nazarov, Superlattices Microstruct. **25**, 1221 (1999).
[15] M. Yu. Kuprianov and V. F. Lukichev, Sov. Phys. JETP **67**, 1163 (1988).
[16] The technical details will be reported elsewhere.
[17] A. Yu. Rusanov *et al.*, Phys. Rev. Lett. **93**, 057002 (2004).
[18] D. Buntinx *et al.*, Phys. Rev. Lett. **94**, 017204 (2005).